These notes likely contain typos and mistakes. This document will be updated with corrections. Last updated: May 10, 2023

Problem 1

a)
The integral measures the amount of gasoline (in gallons) pumped into the tank during the time interval [60, 135] (seconds).
$R_3 = (90-60)\cdot f(90) + (120-90)\cdot f(120) + (135-120)\cdot f(135) = 30\cdot 0.15 + 30\cdot 0.1 + 15\cdot 0.05 = 8.250$ gallons

b)
By Rolle's theorem, since *f(t)* is differentiable on (60, 120), *f(t) is* continuous on [60, 120], and *f(60)=0.1=f(120*), it follows that at least once *f'(c)=0* for some *c* in (60, 120).

c)
$avg\ rate = \frac{1}{(150-0)}\int_0^{150} g(t)\,\mathrm{d}t \approx 0.096\ gallons\ per\ second$ (Graphing calculator)

d)
$g'(140) \approx -0.005\ gallons\ per\ second\ per\ second$ (Graphing calculator)

g'(140) is negative. This means that the rate at which the gasoline flows is decreasing ten seconds before the task of filling the tank ends.

These notes likely contain typos and mistakes. This document will be updated with corrections. Last updated: May 10, 2023

Problem 2

a)

$y''(t) = -2\sin t$

$x''(t) = e^{\cos t}(-\sin t)$

$a(1) = <x''(1), y''(1)> < -e^{\cos 1}\sin 1, -2\sin 1> \approx < -1.444, -1.683>$

b)

$speed = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{e^{2\cos t} + 4\cos^2 t} = 1.5$

$\rightarrow t \approx 1.254$

c)

$$\left(\frac{dy}{dx}\right)_{t=1} = \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)_{t=1} = \frac{2\cos 1}{e^{\cos 1}} \approx 0.630$$

$$x(1) - x(0) = \int_0^1 x'(t)\,\mathrm{d}t$$

$$x(1) = 1 + \int_0^1 e^{\cos t}\mathrm{d}t \approx 3.342$$

d)

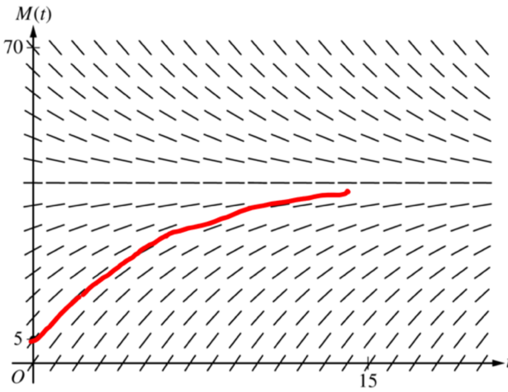
$$distance = \int_0^\pi speed\,\mathrm{d}t = \int_0^\pi \sqrt{e^{2\cos t} + 4\cos^2 t}\,\mathrm{d}t \approx 6.035$$

These notes likely contain typos and mistakes. This document will be updated with corrections. Last updated: May 10, 2023

Problem 3

a)

A solution that passes through the point (0, 5) is shown below.



b)

$$L(t) - M(0) = \frac{dM}{dt}_{t=0}(t-0)$$

$$L(t) - 5 = \frac{1}{4}(40-5)\cdot t$$

$$L(t) = 5 + \frac{35}{4}t$$

$$M(2) \approx L(2) = 5 + \frac{70}{4} = 22.500\ degrees\ Celcius$$

c)

$$\frac{d^2M}{dt^2} = \frac{d}{dt}\left(\frac{dM}{dt}\right) = \frac{d}{dt}\left(10 - \frac{M}{4}\right) = \left(\frac{-1}{4}\right)\frac{dM}{dt} = \left(\frac{-1}{4}\right)\left(\frac{1}{4}(40-M)\right) = \frac{M-40}{16}$$

Since *M(t)* is less than 40 for all values of *t,* the second derivative must be negative, which implies that the solution curve *M(t)* will be concave down. The tangent approximation in part b) is therefore an overestimate.

These notes likely contain typos and mistakes. This document will be updated with corrections. Last updated: May 10, 2023

d)

$$\frac{dM}{dt} = \frac{1}{4}(40 - M)$$

$$\frac{1}{40 - M} dM = \frac{1}{4} dt$$

$$\int \frac{1}{40 - M} dM = \int \frac{1}{4} dt$$

$$-\ln|40 - M| = \frac{t}{4} + C \rightarrow -\ln 35 = C$$

$$\ln|40 - M| = -\frac{t}{4} + \ln(35)$$

$$|40 - M| = 35 \cdot e^{-\frac{t}{4}} \rightarrow 40 - M = \pm 35e^{-\frac{t}{4}} \rightarrow M(t) = 40 - 35e^{-\frac{t}{4}}$$

These notes likely contain typos and mistakes. This document will be updated with corrections. Last updated: May 10, 2023

Problem 4

a)

The graph of *f′* does not change sign at *x=6*, therefore the graph of *f* does not have a relative extremum there.

b)

The graph of *f* will be concave down whenever its second derivative is negative, or whenever the slope of the graph of *f′* is negative. This holds true on the intervals *(-2, 2)* and *(4, 6)*.

c)

Since *f(x)* is continuous, its graph approaches 1 as *x* approaches 2. Since the numerator and denominator both go to zero, we use L'Hospital's Rule below:

$$\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6} = \lim_{x \to 2} \frac{6f'(x) - 3}{2x - 5} = \frac{6 \cdot 0 - 3}{4 - 5} = 3$$

d)

We consider the values of *f(x)* at the endpoints as well as at the critical points in the interior. We use geometry (triangles and quarter circles) and FTC to evaluate the net areas:

$$f(2) = 1$$

$$f(8) = f(2) + \frac{2 \cdot 2}{2} + \left(2 \cdot 4 - \frac{\pi 2^2}{2}\right) = 11 - 2\pi$$

$$f(-1) = f(2) + \frac{3 \cdot 2}{2} = 4$$

$$f(-2) = f(-1) - \frac{1 \cdot 2}{2} = 4 - 1 = 3$$

Using the Closed Interval Method, *f(2)=1* is the absolute minimum.

These notes likely contain typos and mistakes. This document will be updated with corrections. Last updated: May 10, 2023

Problem 5

a)

$$Area = \int_0^3 f(x) - g(x)\, \mathrm{d}x = \int_0^3 f(x)\, dx - \int_0^3 g(x)\, dx$$

$$Area = 10 - \int_0^3 \left(\frac{12}{3+x}\right) \mathrm{d}x = 10 - 12\,(\ln 6 - \ln 3) = 10 - 12 \ln 2$$

b)

$$\int_0^\infty \left(\frac{144}{(3+x)^2}\right) \mathrm{d}x = \lim_{b\to\infty} \int_0^b \left(\frac{144}{(3+x)^2}\right) \mathrm{d}x =$$

$$= \lim_{b\to\infty} \left(-\frac{144}{3+b} + \frac{144}{3}\right) = \frac{144}{3} = 48$$

c)

Integration by parts:

$$\int x f'(x)\, \mathrm{d}x = x f(x) - \int f(x)\, dx$$

$$\int_0^3 x f'(x)\, \mathrm{d}x = 3 f(3) - 0 \cdot f(0) - 10 = 6 - 10 = -4$$

These notes likely contain typos and mistakes. This document will be updated with corrections. Last updated: May 10, 2023

Problem 6

a)

$$f^{(4)}(x) = -2f'\left(x^2\right) - 2xf''\left(x^2\right) \cdot 2x$$

$$f^{(4)}(0) = -2f'(0) = -6$$

$$T_4(x) = 2 + 3x - \frac{2}{2^2}x^2 + \frac{0}{3!}x^3 - \frac{6}{4!}x^4 = 2 + 3x - x^2 - \frac{x^4}{4}$$

b)

$$LEB = \left|\frac{f^{(5)}(z)(x-0)^5}{5!}\right| = \left|\frac{15(0.1-0)^5}{5!}\right| = \frac{1}{8\cdot 10^5} < \frac{1}{10^5}$$

c)

$$g(0) = 4$$

$$g'(x) = e^x f(x)$$

$$g'(0) = e^0 f(0) = 2$$

$$g''(x) = e^x f'(x) + e^x f(x)$$

$$g''(0) = 3 + 2 = 5$$

$$P_2(x) = 4 + 2x + \frac{5}{2!}x^2 = 4 + 2x + \frac{5x^2}{2}$$