AP Calculus BC – FR 2022 | ap-calc.github.io
draft notes (may contain typos and mistakes)

Problem 1

a)

$$\int_{t=1}^{t=5} 450\sqrt{\sin(0.65t)}dt$$

The integral describes the number of vehicles arriving between 6AM and 10AM.

b)

$$\frac{1}{5-1}\int_{t=1}^{t=5} 450\sqrt{\sin(0.65t)}dt = 375.537$$

The average rate from 6AM to 10AM is approximately 375.537 vehicles per hour.

c)

$$A'(t) = \frac{139.5\mathrm{Cos}[0.62t]}{\sqrt{\mathrm{Sin}[0.62t]}}$$

A'(1) = 148.947 > 0 (calculator), therefore the rate A(t) is increasing at 6AM.

d)
Find the value of *a* by solving A(t) = 400 with the aid of a graphing calculator.
Find $a = 1.469$
By the Fundamental Theorem of Calculus, we first find the derivative of N(t) and set it equal to zero or undefined. With technology (TI-84), we find two critical numbers:

$$N'(t) = A(t) - 400 = 0 \text{ or undefined} \rightarrow t = 1.469, t = 3.598$$

N(3.598) = 71.254 → 71 cars is the global maximum on [1.469, 4]. We can justify by the First Derivative Test for Global Extrema: N'(t) changes sign once at t=3.598 from positive to negative, therefore at this point N(t) has a local and global maximum. Alternatively, one can justify using the Closed Interval Method by computing and comparing N(t) values at the endpoints t=1.469 and t=4 with N(3.598).

Problem 2

a)

$$dy/dx = \frac{dy/dt}{dx/dt}_{t=4} = \frac{\ln 18}{\sqrt{17}}$$

= 0.701

b)

$$\text{speed}_{t=4} = \sqrt{(dx/dt)^2 + (dy/dt)^2}_{t=4} = \sqrt{17 + (\ln 18)^2} \approx 5.035$$

$$\text{acceleration}_{t=4} = (x"(4), y"(4)) = (\frac{8}{2\sqrt{17}}, \frac{8}{18}) = (\frac{4}{\sqrt{17}}, \frac{4}{9})$$

c)

$$y(6) - y(4) = \int_{t=4}^{t=6} y'(t)dt \rightarrow y(6) = 5 + \int_{t=4}^{t=6} y'(t)dt \approx 11.571$$

d)

$$\text{Total distance} = \int_{t=4}^{t=6} \sqrt{(dx/dt)^2 + (dy/dt)^2} dy \approx 12.136$$

Problem 3

a)

$$f(4) - f(0) = \int_0^4 f'(x)dx \to f(0) = 3 - \frac{-\pi 2^2}{2} = 3 + 2\pi$$
$$f(5) - f(4) = \int_4^5 f'(x)dx \to f(5) = 3 + \frac{1 * 1}{2} = 7/2$$

b)

The inflection points of *f* occur at x=2 and x=6 because at these two points the second derivative *f''* changes sign, the first derivative *f'* does NOT change sign, and the original function *f* is continuous (since *f* is differentiable).

c)

$$g'(x) = f'(x) - 1 < 0 \to f'(x) < 1$$
$$0 < x < 5$$

d)

$$g'(x) = f'(x) - 1 = 0 \text{ or undefined}$$
$$x = 5 \text{ is a critical number.}$$

We use the Closed Interval Method:

$$g(0) = f(0) - 0 = 3 + 2\pi$$
$$g(5) = f(5) - 5 = 7/2 - 5 = -3/2$$
$$g(7) = f(7) - 7 = 13/2 - 7 = -1/2$$

The absolute minimum equals -3/2 and it occurs at x=5.

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Problem 4

a)

$$r"(8.5) \approx \frac{r'(10) - r'(7)}{10 - 7} = \frac{-3.8 + 4.4}{3} = \frac{1}{5} \text{ cm per day per day}$$

b)
Yes. By the Intermediate Value Theorem, since r'(t) is continuous on [0, 3] and -6 is in between r'(0) and r'(3), there must be a time during the interval (0, 3) such that r'(t) is precisely equal to -6.

c)

$$\int_0^{12} r'(t)dt \approx R_4 = 3f(3) + 4f(7) + 3f(10) + 2f(12) = -15 - 17.6 - 11.4 - 7 = -51cm$$

d)

$$V = \frac{\pi}{3} r^2 h \rightarrow dV/dt = \frac{\pi}{3}(r^2 dh/dt + 2r dr/dt h)$$

$$dV/dt = \frac{\pi}{3}(100^2(-2) + 2 * 100 * (-5.0) * 50) = \frac{-70000\pi}{3} \text{cubic cm per day}$$

Problem 5

a)

$$\text{Area} = \int_1^5 \frac{1}{x} dx = \ln(5) - \ln(1) = \ln(5)$$

b)

$$\text{Volume} = \int_1^5 xe^{x/5} dx = 20e^{1/5}$$

c)

$$\text{Volume} = \pi \int_3^\infty (1/x^2)^2 dx = \pi \lim_{b \to \infty} \int_3^b 1/x^4 dx = ... = \pi/81$$

Problem 6

a)

$$|\frac{a_{n+1}}{a_n}| = |\frac{x^{2n+3}}{2n+3}\frac{2n+1}{x^{2n+1}}| = \frac{2n+1}{2n+3}|x^2| \to x^2 < 1 \to -1 < x < 1$$

$$x = -1 : f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n(-1)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n+1}$$

Convergent; by the Alternating Series Test

$$x = 1 : f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n(1)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$

Convergent; by the Alternating Series Test

Interval of convergence: [-1, 1]

b)

Let $P(x) = x$, the first degree polynomial used to approximate f(x).

$$|f(1/2) - 1/2| = |f(1/2) - P(1/2)| < |\text{next term}| = |\frac{-(1/2)^3}{3}| = \frac{1}{24} < \frac{1}{10}$$

c)

$$f'(x) = 1 - x^2 + x^4 - x^6 + ... + (-1)^n x^{2n} + ...$$

d)

$$f'(x) = \frac{1}{1-(-x^2)} = \frac{1}{1+x^2}$$

$$f'(1/6) = \frac{1}{1+1/36} = \frac{1}{37/36} = \frac{36}{37}$$