Mr. Shubleka' s Key to the 2018
AP Calculus BC Free Response [Draft]

```
(* Problem 1 *)
r[t_] := 44 (t/100)^3 (1 - t/300)^7;

e[t_] := 0.7;

(* Part a *)

Integrate[r[t], {t, 0, 300}] // N

270.
```

270 people enter the line for the escalator during the time interval [0, 300].

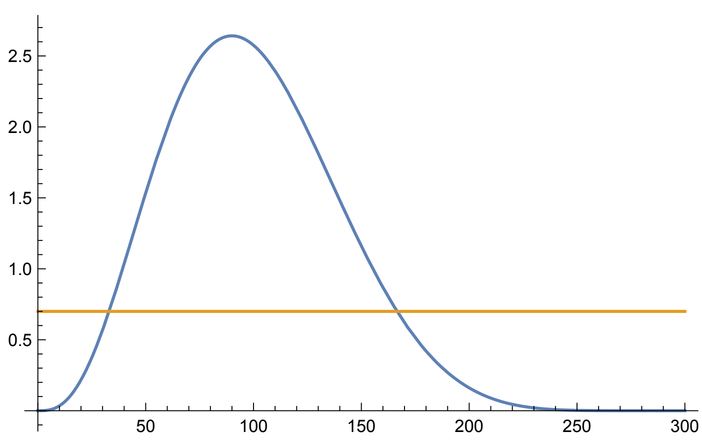
```
(* Part b *)

20 + Integrate[r[t] - e[t], {t, 0, 300}] // N

80.
```

There are 80 people in line for the escalator at t = 300.

```
(* Part c *)

Integrate[e[t], {t, 300, 300 + x}]

0.7 x

Solve[0.7 x == 80, x]

{{x → 114.286}}
```

x = 414.286 seconds.

```
(* Part d *)

Plot[{r[t], e[t]}, {t, 0, 300}]
```




```
Solve[r[t] == e[t], t]
```

```
{t → 33.013297834602746}, {t → 166.57471928287558}

TotalPeople[x_] := 20 + Integrate[r[t] - e[t], {t, 0, x}];

TotalPeople[0]

20

TotalPeople[300]

80.

TotalPeople[33.0133]

3.80344

TotalPeople[166.575]

158.07
```

Using the Closed Interval Method, the minimum number of people is 4,
and this occurs at t = 33.013 seconds.

```
(* Problem 2 *)

p[h_] := 0.2 h^2 E^(-0.0025 h^2);

(* Part a *)

p'[25] // N

-1.17906
```

When the depth is 25 meters, the density of plankton is decreasing at a rate of approximately 1.179 millions of cells per cubic meter per meter.

```
(* Part b *)

Integrate[p[h] * 3, {h, 0, 30}] // N

1675.41
```

There are approximately 1675.410 millions of cells in the 30 meter column.

```
(* Part c *)

Integrate[p[h]  * 3, {h, 0, 30}] + Integrate[u[h] * 3, {h, 30, K}] <=
  1675.410  + 3 * 105 = 1990.40 <= 2000
```

Therefore the number of plankton in this column is less than or equal to 2000 million.

```
(* Part d *)

dx[t_] := 662 Sin[5 t];
dy[t_] := 880 Cos[6 t];

Integrate[Sqrt[(dx[t])^2 + (dy[t])^2], {t, 0, 1}] // N

757.456
```

The boat travels a total of 747.456 meters during the time interval [0, 1].

```
(* Problem 3 *)

g[x_] := 2 (x - 4) ^2;

(* Part a *)

f (1) - Integrate[g[x], {x, -5, 1}] = f (-5)

3 - (3 * (-3) + (-1.5) + 0 + 1)

12.5

f (-5) = 13.

(* Part b* )

4 + Integrate[g[x], {x, 3, 6}]

10

(* Part c *)

f' = g > 0, f'' = g' > 0
```

This occurs on the intervals (0, 1) and (4, 6).

```
(* Part d *)
```

f'' = g' changes sign at x = 4,
hence an inflection point occurs on the graph of f at x = 4.

```
(* Problem 4 *)
(* Part a *)
```

H' (6) is approximately equal to (H (7) - H (5)) / (7 - 2) = 2.500 meters / year.
At t = 6 years,
the height of the tree is increasing at a rate of approximately 2.5 meters per year.

```
(* Part b *)
```

On the interval [2, 5] the average rate of change of H (t) is 2 meters per year. By applying the Mean Value Theorem on H' (t) over the interval [2, 5], we conclude that H' (t), a differentiable function, must have been equal to 2 at least once in (3, 5), an interval that is contained in (2, 10).

```
(* Part c *)
```

1 / (10 - 2) Integrate[H[t], {t, 2, 10}] is approximately equal to: (1 / 8) (1 / 2) (3.5 * 1 + 8 * 2 + 17 * 2 + 26 * 3) = 131.5 / 16 meters (8.219 meters)

```
(* Part d *)

G[x_] := (100 x) / (1 + x);
```

x = 1 meter

```
G'[1] * 0.03
0.75
```

At the time when the tree is 50 meters tall,
the rate of change of the height of the tree is 0.750 meters per year.

```
(* Problem 5 *)

(* Part a *)

2 * 0.5 * Integrate[4^2 - (3 + 2 Cos[theta])^2, {theta, Pi/3, Pi}] OR
0.5 Integrate[4^2 - (3 + 2 Cos[theta])^2, {theta, Pi/3, 5 Pi/3}]

(* Part b *)

y[theta_] := (3 + 2 Cos[theta]) Sin[theta];
x[theta_] := (3 + 2 Cos[theta]) Cos[theta];

(y'[Pi/2]) / (x'[Pi/2])
```

$\frac{2}{3}$

```
(* Part c *)

r[theta_] := 3 + 2 Cos[theta];

r'[theta]
-2 Sin[theta]

r'[Pi/3]
```

$-\sqrt{3}$

Note : dr / dt = dr / dtheta * dtheta / dt

```
3 / (-Sqrt[3])
```

$-\sqrt{3}$

At the instant when the angle is Pi / 3 the angle
is is decreasing at a rate of Sqrt[3] radians per second.

```
(* Problem 6 *)

(* Part a *)

Expand[x (x/3 - ((x/3)^2)/2 + ((x/3)^3)/3 - ((x/3)^4)/4)]
```

$f(x) = \frac{x^2}{3} - \frac{x^3}{18} + \frac{x^4}{81} - \frac{x^5}{324} + \ldots + (-1)^{(n+1)} x^{(n+1)} / (n * 3^n) + \ldots$

```
(* Part b *)

| x + 1 |  < 3
```

Testing the endpoints yields an interval of convergence : (-4, 2]. The series converges at 2 (alternating harmonic) and diverges at x = -4 (harmonic)

```
(* Part c *)
```

ErrorBound = Abs[next term] = $\left|2^5 / 324\right|$ = 32 / 324 = 8/81