Mr. Shubleka's Key to the 2018 AP Calculus AB Free Response [Draft]

```
(* Problem 1 *)
r[t_] := 44 (t/100)^3 (1 - t/300)^7;

e[t_] := 0.7;

(* Part a *)

Integrate[r[t], {t, 0, 300}] // N

270.
```

270 people enter the line for the escalator during the time interval [0, 300].

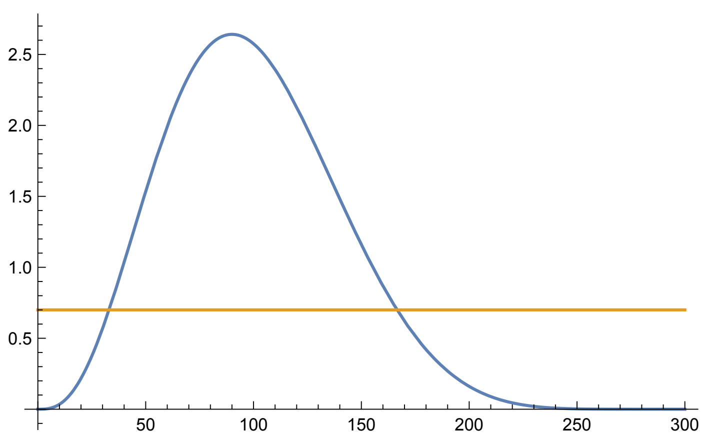
```
(* Part b *)

20 + Integrate[r[t] - e[t], {t, 0, 300}] // N

80.
```

There are 80 people in line for the escalator at t = 300.

```
(* Part c *)

Integrate[e[t], {t, 300, 300 + x}]

0.7 x

Solve[0.7 x == 80, x]

{{x → 114.286}}
```

x = 414.286 seconds.

```
(* Part d *)

Plot[{r[t], e[t]}, {t, 0, 300}]
```




```
Solve[r[t] == e[t], t]
```

```
{t → 33.013297834602746}, {t → 166.57471928287558}

TotalPeople[x_] := 20 + Integrate[r[t] - e[t], {t, 0, x}];

TotalPeople[0]

20

TotalPeople[300]

80.

TotalPeople[33.0133]

3.80344

TotalPeople[166.575]

158.07
```

Using the Closed Interval Method, the minimum number of people is 4,
and this occurs at t = 33.013 seconds.

```
(* Problem 2 *)

v[t_] := (10 Sin[0.4 t^2]) / (t^2 - t + 3);

(* Part a *)

v'[3] // N

-2.1182
```

The acceleration is -2.118.

```
(* Part b *)

-5 + Integrate[v[t], {t, 0, 3}] // N

-1.76021
```

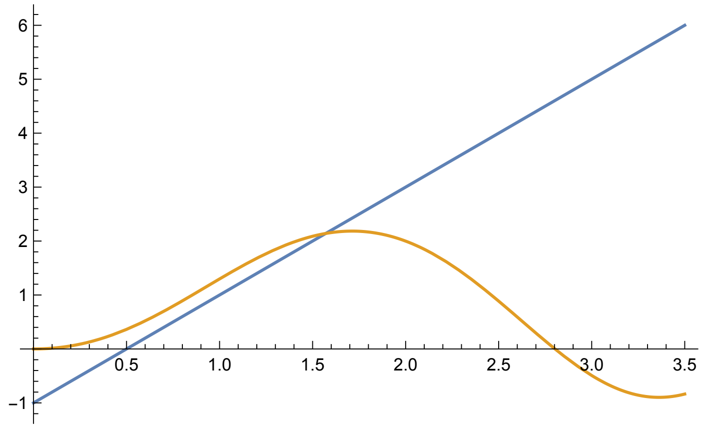
The position at t = 3 is -1.760.

```
(* Part c *)

Integrate[v[t], {t, 0, 3.5}] // N

2.84394

Integrate[Abs[v[t]], {t, 0, 3.5}] // N

3.73708
```

The particle' s displacement during the first 3.5 units of time is 2.844 units of distance.
The particle’s total distance traveled during the first 3.5 units of time is 3.737 units of distance.

```
(* Part d *)
```

```
x2[t_] := t^2 - t;

Plot[{x2'[t], v[t]}, {t, 0, 3.5}]
```



The two particles have the same velocity at t = 1.571 units of time.

```
(* Problem 3 *)

g[x_] := 2 (x - 4) ^2;

(* Part a *)

f (1) - Integrate[g[x], {x, -5, 1}] = f (-5)

3 - (3 * (-3) + (-1.5) + 0 + 1)

12.5

f (-5) = 13.

(* Part b* )

4 + Integrate[g[x], {x, 3, 6}]

10

(* Part c *)

f' = g > 0, f'' = g' > 0
```

This occurs on the intervals (0, 1) and (4, 6).

```
(* Part d *)
```

f'' = g' changes sign at x = 4,
hence an inflection point occurs on the graph of f at x = 4.

```
(* Problem 4 *)
(* Part a *)
```

H' (6) is approximately equal to (H (7) - H (5)) / (7 - 2) = 2.500 meters / year.
At t = 6 years,
the height of the tree is increasing at a rate of approximately 2.5 meters per year.

```
(* Part b *)
```

On the interval [2, 5] the average rate of change of H (t) is 2 meters per year. By applying the Mean Value Theorem on H' (t) over the interval [2, 5], we conclude that H' (t), a differentiable function, must have been equal to 2 at least once in (3, 5), an interval that is contained in (2, 10).

```
(* Part c *)
```

1 / (10 - 2) Integrate[H[t], {t, 2, 10}] is approximately equal to: (1/8) (1/2) (3.5 * 1 + 8 * 2 + 17 * 2 + 26 * 3) = 131.5/16 meters (8.219 meters)

```
(* Part d *)

G[x_] := (100 x) / (1 + x);
```

x = 1 meter

```
G'[1] * 0.03
```

0.75

At the time when the tree is 50 meters tall,
the rate of change of the height of the tree is 0.750 meters per year.

```
(* Problem 5 *)

f[x_] := E^x Cos[x];

(* Part a *)

(f[Pi] - f[0]) / (Pi - 0)
```

$\frac{-1-e^{\pi}}{\pi}$

```
(* Part b *)

Simplify[f'[3 Pi / 2]]
```

$e^{3\pi/2}$

```
(* Part c *)
Factor[f'[x]]
```

e^{x} (Cos[x] - Sin[x])

x → Pi/4, x → 5 Pi/4

```
f[0]
```

1

```
f[Pi]
```

$-e^{\pi}$

```
f[Pi/4]
```

$$\frac{e^{\pi/4}}{\sqrt{2}}$$

```
f[5 Pi/4]
```

$$-\frac{e^{5\pi/4}}{\sqrt{2}}$$

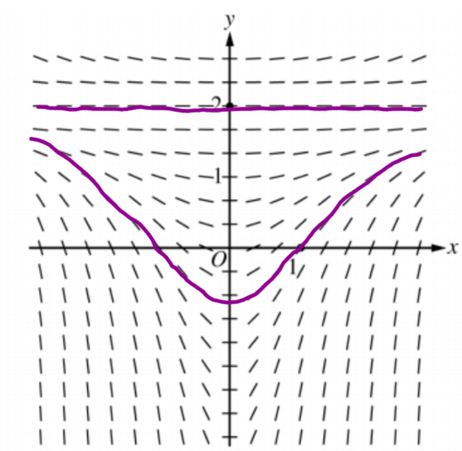
By the Closed Interval Method,
the absolute minimum occurs at x = 5 Pi/4 and is equal to - e^(5 pi/4) /Sqrt[2].

```
(* Part d *)
f'[Pi/2] / 2
```

$$-\frac{e^{\pi/2}}{2}$$

Using L'Hospital's Rule, we conclude that the limit is equal to (-e^(pi/2)) / 2.

```
(* Problem 6 *)
(* Part a *)
```




```
(* Part b *)
```

dy/dx = 4/3 when x = 1 and y = 0

```
L[x_] := 0 + (4/3) (x - 1);
L[0.7]
```

-0.4

f (0.7) is approximately - 0.400.

```
(* Part c *)
Integrate[x/3, x]
```

$$\frac{x^2}{6} + C$$

```
Integrate[1 / (y - 2) ^2, y]
```

$$-\frac{1}{-2 + y}$$

$C \to 1/3$

y - 2 = -6 / (x ^ 2 + 2)

y = 2 - 6 / (x ^ 2 + 2)