Problem 1

a)

L_3 = (2 - 0) 50.3 + (5 - 2)*14.4 + (10 - 5)*6.5=176.300 cubic feet

Using a left hand sum with three subintervals, the volume of the tank is approximately 176.300 cubic feet.

b)

The table suggests that $A(h)$ is a decreasing function, so the left hand sum will be an overestimate of the true volume of the tank.

c)

$$\int_0^{10} f(h)\,dh = \int_0^{10} \frac{50.3}{e^{0.2h}+h}\,dh = 101.325 \text{cubic feet}$$

d)

$$V(h) = V(0) + \int_0^h f(z)\,dz \rightarrow \frac{dV}{dh} = f(h) \quad (by\,FTC)$$

$$\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} = f(h)\frac{dh}{dt} = f(5)*0.26 = 1.694\frac{cf}{\min}$$

When the height of the water is 5 feet, the volume of water is increasing at a rate of 1.694 cubic feet per minute.

Problem 2

a)

$$\int_0^2 f(t)\,dt = \int_0^2 10 + (0.8t)\sin(t^3/100)\,dt = 20.051$$

Approximately 20.051 pounds of bananas were removed during the first two hours.

b)

$$f'(7) = -8.120\frac{pounds}{hour^2}[graphing\ device]$$

$f'(7)$ measures the rate of change of the rate of removal of bananas. The units are bananas per hour*hour. A negative value suggests that the rate of removal is decreasing.

c)

$$g(5) - f(5) = -2.263\ pounds/hr$$

The number of bananas is decreasing at t=5 because the rate of removal is greater than the rate of addition.

d)

$$50 - \int_0^8 f(t)\,dt + \int_3^8 g(t)\,dt = 23.34739574 = 23.347\ pounds$$

At time t=8 there are 23.347 pounds on the display table.

Problem 3

a)

$$f(-2)-f(-6)=\int_{-6}^{-2} f(t)\,dt$$

$$7-f(-6)=\frac{2*4}{2}=4 \rightarrow f(-6)=3$$

$$f(5)-f(-2)=\int_{-2}^{5} f(t)\,dt$$

$$f(5)=3+\left(\frac{3*2}{2}-\frac{\pi 2^2}{2}\right)=6-2\pi$$

b)

f is increasing whenever its derivative is positive. This occurs on the intervals (-6, -2) and (2, 5).

c)

We consider the critical points at x=-2, x=2, as well as the endpoints, which we evaluated in part a).

$$f(-6)=3$$

$$f(-2)=7$$

$$f(2)=7-2\pi$$

$$f(5)=6-2\pi \rightarrow \text{absolute minimum}$$

d)

$$f''(-5)=\frac{-2}{4}=\frac{-1}{2}$$

$$f''(3)=DNE\,(corner)$$

Problem 4

a)

$$H'(0) = \frac{-1}{4}(91-27) = -16$$

$$y - 91 = -16(t-0) \rightarrow y = -16t + 91$$

$$y(3) = -16*3 + 91 = 43$$

Using the tangent line approximation, the temperature at t = 3 is approximately 43 degrees Celcius.

b)

$$H''(t) = \frac{-1}{4}\frac{dH}{dt} = \left(\frac{-1}{4}\right)\left(\frac{-1}{4}\right)(H-27) = \frac{1}{16}(H-27)$$

$H'' > 0$ (since H is always greater than 27)

$\rightarrow$ The graph is concave up, therefore the linear approximation is an underestimate.

c)

$$\frac{dG}{dt} = -(G-27)^{2/3} \quad G(0) = 91$$

$$(G-27)^{-2/3}dG = -1\,dt$$

$$\int (G-27)^{-2/3}dG = \int -1\,dt$$

$$3*(G-27)^{1/3} = -t + C$$

$$3(91-27)^{1/3} = -0 + C \rightarrow C = 12$$

$$G(t) = 27 + \left(\frac{-t+12}{3}\right)^3$$

$G(3) = 54$ degrees celcius.

Problem 5

a)

$x_p' < 0 \to v_p = \frac{2t-2}{t^2-2t+10} = \frac{2(t-1)}{t^2-2t+10} < 0 \to t < 1$

When t is between 0 and 1, particle P is moving to the left because during this time its velocity is negative.

b)

We look for time intervals during which the two velocities have the same sign:

$v_p = \frac{2t-2}{t^2-2t+10} = \frac{2(t-1)}{t^2-2t+10}$

$v_Q = t^2 - 8t + 15 = (t-3)(t-5)$

The particles travel in the same direction for t between 1 and 3 and for t greater than 5.

c)

$a_Q = 2t - 8 \to a_Q(2) = -4$

$v_Q(2) = 3$

The velocity and acceleration have opposite signs; particle Q is slowing

down at t=2, therefore its speed is decreasing.

d)

Particle Q changes direction for the first time at t=3, since:

$v_Q = (t-3)(t-5)$

$x_Q(3) - x_Q(0) = \int_0^3 t^2 - 8t + 15\, dt = \left(\frac{3^3}{3} - 8\frac{3^2}{2} + 15 * 3\right) - (0)$

$x_Q(3) - x_Q(0) = 18 \to x_Q(3) = 5 + 18 = 23$

The position of particle Q at t = 3 is 23 units.

Problem 6

a)

$$f'(x) = -2\sin(2x) + e^{\sin x}\cos x$$

$$f'(\pi) = 0 + 1 * -1 = -1$$

b)

We look for time intervals during which the two velocities have the same sign:

$$k'(x) = h'(f(x))f'(x)$$

$$k'(\pi) = h'(f(\pi))f'(\pi) = h'(2)(-1) = \frac{-1}{3}(-1) = \frac{1}{3}$$

c)

$$m'(x) = -2g'(-2x)h(x) + g(-2x)h'(x)$$

$$m'(2) = -2g'(-4)h(2) + g(-4)h'(2) = -2(-1)\left(\frac{-2}{3}\right) + 5 * \left(\frac{-1}{3}\right) = -3$$

d)

By MVT, there exists a number x=c that satisfies the following, since

g(x) is differentiable (and therefore continuous) on the given interval.

$$g'(c) = \frac{g(-3) - g(-5)}{-3 - (-5)} = \frac{2 - 10}{2} = -4$$