

Problem 1 BC

a)

$$R'(2) \approx \frac{R(3) - R(1)}{3 - 1} = \frac{950 - 1190}{2} = \frac{-240}{2} = -120 \text{ liters / hr}^2$$

b)

$$\int_0^8 R(t) dt \approx 1 * 1340 + 2 * 1190 + 3 * 950 + 2 * 740 = 8050 \text{ liters}$$

The rate removal function is decreasing on the given interval; therefore the left-hand Riemann sum is an overestimate.

c)

$$V(8) - V(0) = \int_0^8 W(t) - R(t) dt = \int_0^8 W(t) dt - \int_0^8 R(t) dt$$

$$V(8) = V(0) + \int_0^8 W(t) dt - \int_0^8 R(t) dt \approx 50,000 + \int_0^8 W(t) dt - 8050 =$$

$$V(8) \approx 50,000 - 7836.195324552195 - 8050 \approx 49786 \text{ liters}$$

d) A graph of $W(t)$ is shown below. Since both rate functions are continuous, then their difference is continuous as well.

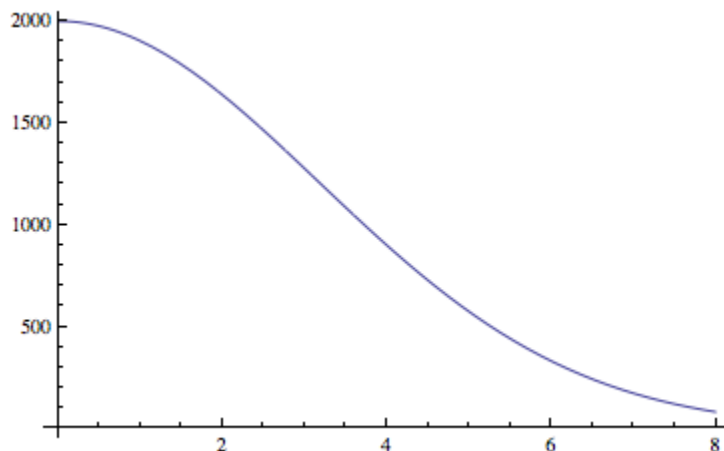
$$W(0) - R(0) < 0$$

$$W(1) - R(1) > 0$$

→ $W(t) - R(t) = 0$ at least once, by IVT.

$$\rightarrow W(t) = R(t)$$

Therefore, the two rates must be equal at least once for some t value on the interval $[0, 8]$.



Problem 2 BC

a)

$$x(3) = x(0) + \int_0^3 x'(t) dt = 5 + \int_0^3 t^2 + \sin(3t^2) dt \approx 14.377$$

$$y(3) = \frac{-1}{2} = -0.500$$

$$(x(3), y(3)) \approx (14.377, -0.500)$$

b)

$$\frac{dy}{dx} \Big|_{t=3} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Big|_{t=3} = \frac{y'(3)}{x'(3)} = \frac{0.5}{9 + \sin(27)} \approx 0.0502191 = 0.050$$

c)

$$\text{speed}_{t=3} = \sqrt{(x'(3))^2 + (y'(3))^2} \approx 9.96892 \approx 9.969$$

d)

$$\begin{aligned} \text{Distance}_{[0,2]} &= \int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2} dt = \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2} dt + \int_1^2 \sqrt{(x'(t))^2 + (y'(t))^2} dt \\ &= \int_0^1 \sqrt{(x'(t))^2 + (-2)^2} dt + \int_1^2 \sqrt{(x'(t))^2 + (0)^2} dt = 4.34987 \approx 4.350 \end{aligned}$$

Problem 3 BC

a)

$$g'(x) = f(x)$$

At $x = 10$, the derivative of $g(x)$ does not change sign (it stays negative), therefore $g(10)$ is neither a relative maximum nor a relative minimum.

b)

At $x = 4$, the first derivative of $g(x)$ does not change sign (it stays positive), and its second derivative ($g''(x) = f'(x)$) changes from positive to negative; therefore $g(x)$ has an inflection point at $x = 4$.

c)

Critical Numbers:

$$g'(x) = f(x) = 0 \text{ or undefined} \rightarrow x = -2, 2, 6, 10$$

$$g(-2) = -8$$

$$g(2) = 0$$

$$g(6) = 8$$

$$g(10) = 0$$

Endpoints:

$$g(-4) = -4$$

$$g(12) = -4$$

By the Closed Interval Method, the absolute maximum and minimum are -8 and 8 , respectively. They occur at points $(-2, -8)$ and $(6, 8)$.

d)

$$g(x) \leq 0 \rightarrow 10 \leq x \leq 12 \text{ and } -4 \leq x \leq 2$$

Net areas are zero or negative for these x -values, by FTC.

Problem 4 BC

$$\frac{dy}{dx} = x^2 - \frac{1}{2}y$$

a)

$$\frac{dy}{dx} = x^2 - \frac{1}{2}y$$

$$\frac{d^2y}{dx^2} = 2x - \frac{1}{2} \frac{dy}{dx} = 2x - \frac{1}{2} \left(x^2 - \frac{1}{2}y \right) = 2x - \frac{x^2}{2} + \frac{y}{4}$$

b)

$$\frac{dy}{dx} \Big|_{x=-2, y=8} = 4 - 4 = 0$$

$$\frac{d^2y}{dx^2} \Big|_{x=-2, y=8} = -4 - 2 + 2 = -4 < 0$$

By the Second Derivative Test for Local Extrema, the solution function will have a local maximum at $(-2, 8)$.

c)

We use L'Hospital's Rule twice below.

$$g(-1) = 2$$

$$\lim_{x \rightarrow -1} \left(\frac{g(x) - 2}{3(x+1)^2} \right) = \lim_{x \rightarrow -1} \left(\frac{g'(x)}{6(x+1)} \right) = \lim_{x \rightarrow -1} \left(\frac{x^2 - \frac{1}{2}y}{6(x+1)} \right) = \lim_{x \rightarrow -1} \left(\frac{2x - \frac{1}{2} \frac{dy}{dx}}{6} \right) =$$

$$\lim_{x \rightarrow -1} \left(\frac{2x - \frac{x^2}{2} + \frac{y}{4}}{6} \right) = \frac{-2 - \frac{1}{2} + \frac{1}{2}}{6} = \frac{-1}{3}$$

d)

$$h(0) = 2$$

$$\Delta x = 0.5$$

$$y - 2 = \frac{dy}{dx} \Big|_{x=0} (x - 0) \rightarrow y = 2 + (-1) * 0.5 = 1.5$$

$$y - 1.5 = \frac{dy}{dx} \Big|_{x=0.5} (x - 0.5) \rightarrow y = 1.5 + (-0.5) * 0.5 = 1.25$$

$$h(1) \approx 1.250$$

Problem 5 BC

a)

$$r_{\text{avg}[0,10]} = \frac{1}{10-0} \int_0^{10} \frac{1}{20} (3+h^2) dh = \frac{1}{200} \int_0^{10} (3+h^2) dh = \frac{1}{200} \left(\left(3h + \frac{h^3}{3} \right)_{h=10} - \left(3h + \frac{h^3}{3} \right)_{h=0} \right) = \frac{109}{60}$$

b)

$$\begin{aligned} V &= \pi \int_0^{10} \left(\frac{1}{20} (3+h^2) \right)^2 dh = \frac{\pi}{400} \int_0^{10} (3+h^2)^2 dh = \frac{\pi}{400} \left(\left(9h + 2h^3 + \frac{h^5}{5} \right)_{h=10} - \left(9h + 2h^3 + \frac{h^5}{5} \right)_{h=0} \right) = \\ &= \frac{\pi}{400} \left(90 + 2000 + \frac{100000}{5} \right) = \frac{\pi}{40} (9 + 200 + 2000) = \frac{2209\pi}{40} \end{aligned}$$

c)

$$r = \frac{1}{20} (3+h^2)$$

$$\frac{dr}{dt} \Big|_{h=3 \text{ inches}} = \frac{-1}{5}$$

$$\frac{dr}{dt} = \frac{1}{20} 2h \frac{dh}{dt} \rightarrow \frac{-1}{5} = \frac{h}{10} \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{-2}{h} = \frac{-2}{3} \text{ inches / second.}$$

Problem 6 BC

a)

$$f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \dots + \frac{f^{(n)}(1)}{n!}(x-1)^n + \dots$$

$$f(x) = 1 - \frac{1}{2}(x-1) + \frac{1/4}{2}(x-1)^2 - \frac{2/8}{6}(x-1)^3 + \dots + (-1)^n \frac{(n-1)!/2^n}{n!}(x-1)^n + \dots$$

$$f(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{8}(x-1)^2 - \frac{1}{24}(x-1)^3 + \dots + (-1)^n \frac{1}{n 2^n}(x-1)^n + \dots$$

b)

The center is $x = 1$; with a radius of 2, we need to test the endpoints at $x = -1$ and $x = 3$.

$$f(x) = \sum \frac{(-1)^n}{n 2^n} (x-1)^n$$

$$x = -1 \rightarrow \sum \frac{(-1)^n}{n 2^n} (-1-1)^n = \sum \frac{1}{n 2^n} (2)^n = \sum \frac{1}{n} \text{ (Harmonic Series; Diverges)}$$

$$x = 3 \rightarrow \sum \frac{(-1)^n}{n 2^n} (3-1)^n = \sum \frac{(-1)^n}{n} \text{ (Alternating Harmonic Series; Converges)}$$

The interval of convergence is $(-1, 3]$.

c)

$$f(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{8}(x-1)^2 - \frac{1}{24}(x-1)^3 + \dots + (-1)^n \frac{1}{n 2^n}(x-1)^n + \dots$$

$$f(1.2) \approx 1 - \frac{1}{2}(1.2-1) + \frac{1}{8}(1.2-1)^2 = 0.905$$

d)

For Alternating Series, the error bound is the absolute value of the next term:

$$\left| -\frac{1}{24}(1.2-1)^3 \right| = \frac{1}{24} \frac{1}{5^3} = \frac{1}{3000} < 0.001$$

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