Questions at: http://www.ilearnmath.net/help/index.php?page=apfreeresponse

Problem 1

a)

$G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$

$G'(t) = -45\sin\left(\frac{t^2}{18}\right)\frac{2t}{18} = -5t\sin\left(\frac{t^2}{18}\right)$

$G'(5) = -25\sin\left(\frac{25}{18}\right) = -24.588 \ \frac{tons/hr}{hr}$

The rate at which unprocessed gravel arrives at the gravel processing plant is decreasing at a rate of 24.588 tons/hr^2 five hours into the workday.

b)

Net Change$=\int_0^8 G(t)\,dt = \int_0^8 90 + 45\cos\left(\frac{t^2}{18}\right)dt = 825.551$ tons of unprocessed gravel

arrives at the plant during the workday.

c)

$G(5) - R(5) = 90 + 45\cos\left(\frac{25}{18}\right) - 100 = 98.141 - 100 = -1.859\frac{tons}{hr} < 0$

Therefore, the total amount is decreasing five hours into the workday.

d)

We must use the closed interval method, since the domain is a closed interval:

$T(0) = 500$

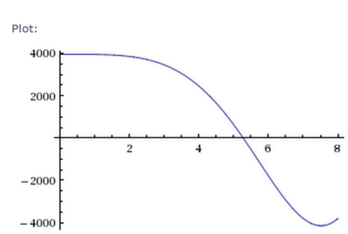
$T(8) = T(0) + \int_0^8 G(t) - 100\,dt = 500 + 825.441 = 525.551$

$G(t)$ changes sign from positive to negative once [t=5.684], hence we measure:

T(5.684)=$T(0) + \int_0^{5.684} G(t) - 100\,dt = 627.593$

Maximum $= T(5.684) = 627.593$ tons, occurs at $t = 5.684$.

Plot of $G(t)$:



Problem 2

a)

$$Area = \pi r_{r=3}^2 - 2 * \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}\left(r_{far}^2 - r_{near}^2\right) d\theta = 9\pi - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(3^2 - \left(4 - 2\sin\theta\right)^2\right) d\theta =$$

$$= 9\pi - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(3^2 - \left(4 - 2\sin\theta\right)^2\right) d\theta = 24.709[TI - 84]$$

b)

$$x = r\cos\theta = (4 - 2\sin\theta)\cos\theta$$

$$x(t) = (4 - 2\sin\theta)\cos\theta = \left(4 - 2\sin\left(t^2\right)\right)\cos\left(t^2\right) = -1$$

$$TI - 84 : t = 1.428$$

c)

position vector =

$$\left(x(t), y(t)\right) = \left(r\cos t^2, r\sin t^2\right) = \left((4 - 2\sin t^2)\cos t^2, (4 - 2\sin t^2)\sin t^2\right)$$

velocity vector =

$$\left(x'(1.5), y'(1.5)\right) = (-8.072, -1.673)$$

Problem 3

a)

$$C'(5) \approx \frac{C(4)-C(3)}{4-3} = \frac{12.8-11.2}{1} = 1.600 \frac{\text{ounces}}{\text{min}}$$

b)

Yes. The Mean Value Theorem suggests that, since the function is differentiable (and therefore continuous, on the given interval, then at least once, we must have:

$$C'(t) = \frac{C(4)-C(2)}{4-2} = \frac{12.8-8.8}{2} = \frac{4}{2} = 2\frac{\text{ounces}}{\text{min}}, \text{ for some } t \text{ in } (2,4).$$

c)

$$\frac{1}{6}\int_0^6 C(t)\,dt \approx \frac{1}{6}M_3 = \frac{1}{6}\left(f(1)*2+f(3)*2+f(5)*2\right) = \frac{2}{6}\left(5.3+11.2+13.8\right)$$

$$= \frac{30.3}{3} = 10.100 \text{ ounces}$$

is the average amount of coffee in the cup during the interval $[0, 6]$.

d)

$$B(t) = 16-16e^{-0.4t}$$

$$B'(t) = (-16)*(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$$

$$B'(5) = 6.4e^{-2} = \frac{6.4}{e^2}\frac{\text{ounces}}{\text{min}}$$

Problem 4

a)
The function f has a local minimum whenever its derivative changes sign from negative to positive. This occurs once, at $x = 6$.

b)
Applying FTC on the entire interval and using the information about the given areas, we have:

$$f(8) - f(0) = \int_0^8 f'(x)\,dx$$

$$f(0) = f(8) - \int_0^8 f'(x)\,dx = 4 - (2 + 6 - 3 + 7) = -8$$

Using net areas, we find and summarize:

$f(1) = -8 + 2 = -6$

$f(4) = -6 + 6 = 0$

$f(6) = 0 - 3 = -3$

$f(8) = 4$

$f(0) = -8$

By the Closed Interval Method, the absolute minimum is -8, and it occurs at the endpoint $x = 0$.

c)
The graph of f is increasing whenever the first derivative is positive, and concave down whenever the slope of the first derivative is negative. (Note that the slope of the first derivative is simply the second derivative.) These two properties are satisfied on the intervals (0, 1) and (3, 4).

d)

$$g(x) = \left(f(x)\right)^3$$

$$g'(x) = 3\left(f(x)\right)^2 f'(x)$$

$$g'(3) = 3\left(f(3)\right)^2 f'(3) = 3 * \left(\frac{-5}{2}\right)^2 * 4 = 75$$

The slope is 75.

Problem 5

a)

We use L'Hopital's Rule here:

$$\lim_{x\to 0}\frac{f(x)+1}{\sin x}=\lim_{x\to 0}\frac{f'(x)}{\cos x}=\lim_{x\to 0}\frac{y^2(2x+2)}{\cos x}=\frac{2}{1}=2$$

b)

$$\Delta x = 0.25$$

$$x=0, y=-1, m=2 ::: y-(-1)=2(x-0)\to y=2x-1\to y(0.25)=-0.5$$

$$x=0.25, y=-0.5, m=\frac{5}{8} ::: y-(-0.5)=\frac{5}{8}(x-0.25)\to y(0.5)=\frac{5}{32}-\frac{1}{2}=\frac{-11}{32}$$

$$f(0.5)\approx\frac{-11}{32}$$

c)

$$\frac{dy}{dx}=y^2(2x+2)$$

$$y^{-2}dy=(2x+2)dx$$

$$\int y^{-2}\,dy=\int(2x+2)\,dx$$

$$\frac{-1}{y}=x^2+2x+C\to\frac{-1}{-1}=C\to C=1$$

$$\frac{-1}{y}=x^2+2x+1$$

$$y=f(x)=\frac{-1}{x^2+2x+1}$$

Problem 6

a)

$$P_1(x) = f(0) + f'(0)(x-0) = -4 + f'(0)x$$

$$P_1(0.5) = -4 + f'(0)*0.5 = -3 \rightarrow f'(0) = \frac{-3+4}{0.5} = 2$$

b)

$$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$P_3(x) = -4 + 2x + \frac{-2/3}{2!}x^2 + \frac{1/3}{3!}x^3$$

$$P_3(x) = -4 + 2x - \frac{1}{3}x^2 + \frac{1}{18}x^3$$

c)

$$h'(x) = f(2x)$$

$$T_3(x) = h(0) + h'(0)x + \frac{h''(0)}{2!}x^2 + \frac{h'''(0)}{3!}x^3$$

$$h(0) = 7$$

$$h'(x) = f(2x) \rightarrow h'(0) = f(2*0) = -4$$

$$h''(x) = 2f'(2x) \rightarrow h''(0) = 2f'(2*0) = 2*2 = 4$$

$$h'''(x) = 4f''(2x) \rightarrow h'''(0) = 4f''(2*0) = 4*\frac{-2}{3} = \frac{-8}{3}$$

$$T_3(x) = 7 - 4x + \frac{4}{2!}x^2 + \frac{-8/3}{3!}x^3 = 7 - 4x + 2x^2 - \frac{4}{9}x^3$$