

Please find Questions at:

<http://www.ilearnmath.net/help/index.php?page=apfreerresponse>**Problem 1**

a)

$$G(t) = 90 + 45 \cos\left(\frac{t^2}{18}\right)$$

$$G'(t) = -45 \sin\left(\frac{t^2}{18}\right) \frac{2t}{18} = -5t \sin\left(\frac{t^2}{18}\right)$$

$$G'(5) = -25 \sin\left(\frac{25}{18}\right) = -24.588 \frac{\text{tons/hr}}{\text{hr}}$$

The rate at which unprocessed gravel arrives at the gravel processing plant is decreasing at a rate of 24.588 tons/hr<sup>2</sup> five hours into the workday.

b)

Net Change =  $\int_0^8 G(t) dt = \int_0^8 90 + 45 \cos\left(\frac{t^2}{18}\right) dt = 825.551$  tons of unprocessed gravel arrives at the plant during the workday.

c)

$$G(5) - R(5) = 90 + 45 \cos\left(\frac{25}{18}\right) - 100 = 98.141 - 100 = -1.859 \frac{\text{tons}}{\text{hr}} < 0$$

Therefore, the total amount is decreasing five hours into the workday.

d)

We must use the closed interval method, since the domain is a closed interval:

$$T(0) = 500$$

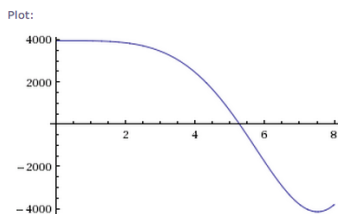
$$T(8) = T(0) + \int_0^8 G(t) - 100 dt = 500 + 825.441 = 525.551$$

$G(t)$  changes sign from positive to negative once [ $t=5.684$ ], hence we measure:

$$T(5.684) = T(0) + \int_0^{5.684} G(t) - 100 dt = 627.593$$

Maximum =  $T(5.684) = 627.593$  tons, occurs at  $t = 5.684$ .

Plot of  $G(t)$ :



**Problem 2**

a)

$$\text{speed} = 2 \rightarrow |v(t)| = 2$$

$$|(-2 + (t^2 + 3t)^{6/5} - t^3)| = 2$$

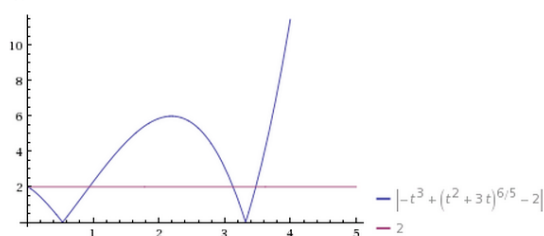
The speed of the particle equals 2 at two different times in the given interval:

$$t = 3.128$$

$$t = 3.473$$

[Using the intersection feature on the TI graphing calculator, and rounding to three decimal places.]

Plot:



b)

$$s(t) - s(0) = \int_{w=0}^{w=t} v(w) dw$$

$$s(t) = s(0) + \int_{w=0}^{w=t} v(w) dw = 10 + \int_0^t -2 + (w^2 + 3w)^{6/5} - w^3 dw$$

$$s(5) = 10 + \int_0^5 -2 + (w^2 + 3w)^{6/5} - w^3 dw = -9.207$$

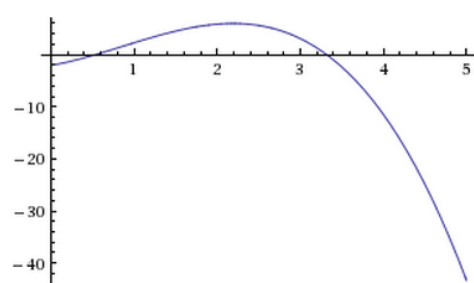
c)

The particle changes direction whenever the velocity function changes sign. This occurs twice:

$$t = 0.536$$

$$t = 3.318$$

Plot:



d)

From the graph of speed in part a), we observe that speed is increasing at  $t = 4$ .

OR

From the graph of velocity in part c), we observe that the velocity is negative at this time, and acceleration, which is the slope to the velocity graph, is also negative.

Speed is increasing whenever velocity and acceleration have the same sign.

OR

(Note that we could also directly compute to confirm that  $v(4) < 0, a(4) < 0$  using the calculator.)

**Problem 3**

a)

$$C'(5) \approx \frac{C(4) - C(3)}{4 - 3} = \frac{12.8 - 11.2}{1} = 1.600 \frac{\text{ounces}}{\text{min}}$$

b)

Yes. The Mean Value Theorem suggests that, since the function is differentiable (and therefore continuous, on the given interval, then at least once, we must have:

$$C'(t) = \frac{C(4) - C(2)}{4 - 2} = \frac{12.8 - 8.8}{2} = \frac{4}{2} = 2 \frac{\text{ounces}}{\text{min}}, \text{ for some } t \text{ in } (2, 4).$$

c)

$$\begin{aligned} \frac{1}{6} \int_0^6 C(t) dt &\approx \frac{1}{6} M_3 = \frac{1}{6} (f(1) * 2 + f(3) * 2 + f(5) * 2) = \frac{2}{6} (5.3 + 11.2 + 13.8) \\ &= \frac{30.3}{3} = 10.100 \text{ ounces} \end{aligned}$$

is the average amount of coffee in the cup during the interval  $[0, 6]$ .

d)

$$B(t) = 16 - 16e^{-0.4t}$$

$$B'(t) = (-16) * (-0.4)e^{-0.4t} = 6.4e^{-0.4t}$$

$$B'(5) = 6.4e^{-2} = \frac{6.4}{e^2} \frac{\text{ounces}}{\text{min}}$$

**Problem 4**

a)

The function  $f$  has a local minimum whenever its derivative changes sign from negative to positive. This occurs once, at  $x = 6$ .

b)

Applying FTC on the entire interval and using the information about the given areas, we have:

$$f(8) - f(0) = \int_0^8 f'(x) dx$$

$$f(0) = f(8) - \int_0^8 f'(x) dx = 4 - (2 + 6 - 3 + 7) = -8$$

Using net areas, we find and summarize:

$$f(1) = -8 + 2 = -6$$

$$f(4) = -6 + 6 = 0$$

$$f(6) = 0 - 3 = -3$$

$$f(8) = 4$$

$$f(0) = -8$$

By the Closed Interval Method, the absolute minimum is  $-8$ , and it occurs at the endpoint  $x = 0$ .

c)

The graph of  $f$  is increasing whenever the first derivative is positive, and concave up whenever the slope of the first derivative is positive. (Note that the slope of the first derivative is simply the second derivative.) These two properties are satisfied on the intervals  $(1, 3)$  and  $(6, 8)$ .

d)

$$g(x) = (f(x))^3$$

$$g'(x) = 3(f(x))^2 f'(x)$$

$$g'(3) = 3(f(3))^2 f'(3) = 3 * \left(\frac{-5}{2}\right)^2 * 4 = 75$$

The slope is 75.

**Problem 5**

a)

$$\begin{aligned} \text{Area} &= \int_0^2 g(x) - f(x) dx = \int_0^2 4 \cos\left(\frac{1}{4}\pi x\right) - (2x^2 - 6x + 4) dx = \\ &= \int_0^2 4 \cos\left(\frac{1}{4}\pi x\right) dx - \int_0^2 (2x^2 - 6x + 4) dx = \\ &= \frac{16}{\pi} \sin\left(\frac{1}{4}\pi x\right) \Big|_0^2 - \left(\frac{2x^3}{3} - 3x^2 + 4x\right) \Big|_0^2 = \frac{16}{\pi} - \frac{4}{3} \text{ square units} \end{aligned}$$

b)

We use the Washer Method:

$$\begin{aligned} V &= \pi \int_0^2 R^2(x) - r^2(x) dx = \pi \int_0^2 (4 - f(x))^2 - (4 - g(x))^2 dx \\ V &= \pi \int_0^2 \left(4 - (2x^2 - 6x + 4)\right)^2 - \left(4 - \left(4 \cos\left(\frac{1}{4}\pi x\right)\right)\right)^2 dx \end{aligned}$$

c)

$$A(x) = (g(x) - f(x))^2$$

$$V = \int_0^2 A(x) dx = \int_0^2 (g(x) - f(x))^2 dx = \int_0^2 \left(4 \cos\left(\frac{1}{4}\pi x\right) - (2x^2 - 6x + 4)\right)^2 dx$$

**Problem 6**

a)

$$\text{slope} = \frac{dy}{dx}_{(1,0)} = e^y(3x^2 - 6x)_{(1,0)} = e^0(3 - 6) = -3$$

$$y - 0 = -3(x - 1) \rightarrow y = -3x + 3$$

$$f(1.2) \approx y(1.2) = -3(1.2) + 3 = -3.6 + 3 = -0.6$$

b)

$$\frac{dy}{dx} = e^y(3x^2 - 6x)$$

$$e^{-y} dy = (3x^2 - 6x) dx$$

$$\int e^{-y} dy = \int (3x^2 - 6x) dx$$

$$-e^{-y} = x^3 - 3x^2 + C$$

$$e^{-y} = -x^3 + 3x^2 + K \rightarrow K = 1 + 1 - 3 = -1$$

$$e^{-y} = -x^3 + 3x^2 - 1$$

$$-y = \ln |-x^3 + 3x^2 - 1|$$

$$y = f(x) = \ln \left( \frac{1}{|-x^3 + 3x^2 - 1|} \right)$$