Problem 1

a)

$$W'(12) \approx \frac{W(15)-W(9)}{15-9} = \frac{67.9-61.8}{6} = \frac{6.1}{6} \approx 1.017 \frac{\text{degrees Fahrenheit}}{\text{minute}}$$

At $t = 12\text{minutes}$, we estimate that the temperature of water is increasing at a rate approximately equal to the average rate of change between 9 and 15 minutes, which is 1.017 degrees Fahrenheit per minute. (Answer rounded to three decimal places.)

b)

$\int_0^{20} W'(t)dt = W(20) - W(0) = 71.0 - 55.0 = 16 \text{ degrees Fahrenheit}$. This is the Net Change in the temperature of water from 0 to 20 minutes.

c)

$$L_4 = W(0)(44-0) + W(4)(9-5) + W(9)(15-9) + W(15)(20-15) =$$
$$L_4 = 55*4 + 57.1*5 + 61.8*6 + 67.9*5 = 1215.8$$
$$\frac{1}{20}\int_0^{20} W(t)dt \approx \frac{1}{20}L_4 = \frac{1215.8}{20} = 60.790 \text{ degrees Fahrenheit}$$

The approximate average temperature over the 20 minutes is 60.790 degrees Fahrenheit.

This is an underestimate because the function used as a height of the rectangles in the Riemann Sum is increasing; hence the resulting rectangles fail to capture the entire area under the graph.

d)

$$W(25) - W(20) = \int_{20}^{25} W'(t)dt = 2.04315\ (TI-84)$$
$$W(25) = W(20) + 2.04315 = 71.0 + 2.04315 = 73.043 \text{ deg Fahrenheit}$$

Problem 2

a)

$$\left.\frac{dx}{dt}\right|_{t=2} = \frac{\sqrt{2+2}}{e^2} = \frac{2}{e^2} > 0 \rightarrow \text{ moving to the right}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin^2 2}{\frac{\sqrt{4}}{e^2}} = \frac{(e\sin 2)^2}{2} = 3.055 = slope$$

b)

By FTC, $x(4) - x(2) = \int_2^4 x'(t)dt \rightarrow x(4) = x(2) + 0.252954 = 1 + 0.252954 = 1.253$

c)

$$\text{speed } = \left.\sqrt{\left(\tfrac{dx}{dt}\right)^2 + \left(\tfrac{dy}{dt}\right)^2}\right|_{t=4} = 0.575\,[TI-84]$$

$x''(4) = -0.0411253 \approx -0.041 \quad [TI-84]$

$y''(4) = 0.989358 \approx 0.989 \quad [TI-84]$

acceleration vector = $\langle x''(4), y''(4)\rangle = \langle -0.041, 0.989\rangle \quad [TI-84]$

d)

Distance traveled =

$$\int_{t=2}^{t=4} \sqrt{\left(\tfrac{dx}{dt}\right)^2 + \left(\tfrac{dy}{dt}\right)^2}\,dt = \int_2^4 \sqrt{\left(\tfrac{\sqrt{t+2}}{e^t}\right)^2 + \left(\sin^2 t\right)^2}\,dt = \int_2^4 \sqrt{\tfrac{t+2}{e^{2t}} + \sin^4 t}\,dt = 0.650983 \text{ [TI-84]}$$

The distance traveled during the interval [2, 4] is approximately 0.651 units.

Problem 3

a)

$$g(2)=\int_1^2 f(t)dt=\tfrac{1}{2}*base*height=\tfrac{1}{2}*1*\tfrac{-1}{2}=\tfrac{-1}{4}$$

$$g(-2)=\int_1^{-2} f(t)dt=-\int_{-2}^{1} f(t)dt=-[(right\ triangle)-(semicircle)]=-[\tfrac{3}{2}-\tfrac{\pi}{2}]=\tfrac{\pi-3}{2}$$

Note that the minus sign in front of 'semicircle' is added because the semi-circle lies below the horizontal axis, whereas the right triangle lies above.

b)

$g'(x)=f(x), g''(x)=f'(x)$

$g'(-3)=f(-3)=2$

$g''(-3)=f'(-3)=1$ (read slope from the graph)

c)

$g'(x)=0\rightarrow f(x)=0\rightarrow x=-1,1$

$x=-1\rightarrow g'(x)$, which is $f(x)$, changes from positive to negative

$g(-1)$ relative maximum

x=1 $\rightarrow g'(x)$, which is $f(x)$, does not change sign,but $g''(x)$ does.

$g(1)$ is not a relative extremum. It is an inflection point.

d)

An inflection point occurs where $g(x)$ is continuous and $g''(x)$ changes sign but $g'(x)$ does not.

In the context of this problem, we are looking for points where $f'(x)$ changes sign, yet $f(x)$ maintains its sign. This occurs at (-2, 3), (0, -1), and (1, 0). These are the three inflection points.

Problem 4

a)

$$L(x) = f(1) + f'(1)(x-1) = 15 + 8(x-1)$$
$$y = 8x + 7$$
$$f(1.4) \approx L(1.4) = 8*1.4 + 7 = 11.2 + 7 = 18.200$$

b)

$$\int_1^{1.4} f'(x)dx = f(1.1)*(1.2-1) + f(1.3)*(1.4-1.2) = (10+13)*0.2 = 4.600$$
$$f(1.4) - f(1) = \int_1^{1.4} f'(x)dx$$
$$f(1.4) = f(1) + \int_1^{1.4} f'(x)dx \approx 15 + 4.600 = 19.600$$

c)

x	y	$\frac{dy}{dx}$	Computations
1	15	8	L(1.2) = 15 + 8 (1.2 – 1) = 15 + 1.6 = 16.6
1.2	16.6	12	L(1.4) = 16.6 + 12 (1.4 – 1.2) =16.6 + 2.4= 19
1.4	**19.000**		

By Euler's Method, $f(1.4) \approx 19.000$.

d)

$$T_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 = 15 + 8(x-1) + \frac{20}{2}(x-1)^2$$
$$T_2(x) = 15 + 8x - 8 + 10(x^2 - 2x + 1) = 10x^2 - 12x + 17$$
$$f(1.4) \approx T_2(1.4) = 10*1.4^2 - 12*1.4 + 17 = 19.800$$

Problem 5

a)

$$\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(100-40) = 12 \; grams/day$$
$$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(100-70) = 6 \; grams/day$$

The bird is gaining weight faster when it weighs 40 grams.

b)

$$\frac{d^2B}{dt^2} = \frac{d}{dt}\left(\frac{dB}{dt}\right) = \frac{d}{dt}\left(\frac{1}{5}(100-B)\right) = \frac{-1}{5}\frac{dB}{dt} = \frac{-1}{5}\frac{1}{5}(100-B) = \frac{-1}{25}(100-B)$$

When B is less than 100, the second derivative suggests that the graph must be concave down for all values of B in this range. The given graph is concave up at first, and then concave down.

c)

$$\frac{dB}{dt} = \frac{1}{5}(100-B)$$
$$\frac{1}{100-B}\,dB = \frac{1}{5}\,dt$$
$$\int \frac{1}{100-B}\,dB = \int \frac{1}{5}\,dt$$
$$-\int \frac{-1}{100-B}\,dB = \int \frac{1}{5}\,dt$$
$$-\ln|100-B| = \frac{t}{5} + C$$
$$B(0) = 20 \rightarrow -\ln 80 = C$$
$$-\ln|100-B| = 0.2t - \ln 80$$
$$\ln|100-B| = \ln 80 - 0.2t$$
$$100 - B = e^{\ln 80 - 0.2t}$$
$$B(t) = 100 - 80e^{-0.2t}$$

Problem 6

a)

Ratio Test

$$a_n = \frac{(-1)^n x^{2n+1}}{2n+3} \quad a_{n+1} = \frac{(-1)^{n+1} x^{2(n+1)+1}}{2(n+1)+3} = \frac{(-1)^{n+1} x^{2n+3}}{2n+5}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1}x^{2n+3}}{2n+5}}{\frac{(-1)^n x^{2n+1}}{2n+3}}\right| = \lim_{n\to\infty}\left|(-1)\frac{\frac{x^{2n+3}}{2n+5}}{\frac{x^{2n+1}}{2n+3}}\right| = \lim_{n\to\infty}\left|\frac{x^{2n+3}}{2n+5}\frac{2n+3}{x^{2n+1}}\right| = \lim_{n\to\infty}\left|x^2\frac{2n+3}{2n+5}\right| = \left|x^2\right| = x^2 < 1$$

$$\Rightarrow -1 < x < 1$$

Test Endpoints:

$$x=-1 \Rightarrow \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+3} = \sum_{n=0}^{\infty}(-1)^n\frac{(-1)^{2n+1}}{2n+3} = \sum_{n=0}^{\infty}\frac{(-1)^{2n+n+1}}{2n+3} = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{2n+3}$$

Alternating Series Test -- Converges

$$x=1 \Rightarrow \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+3} = \sum_{n=0}^{\infty}(-1)^n\frac{(1)^{2n+1}}{2n+3} = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+3}$$

Alternating Series Test -- Converges

Interval of Convergence: $[-1,1]$.

b)

It's an alternating series, so the error will be less than the absolute value of the next term:

$$\left|g(\tfrac{1}{2}) - T_2(\tfrac{1}{2})\right| \le \left|\frac{1}{7}x^5\right|_{x=\frac{1}{2}} = \left|\frac{1}{7}\left(\tfrac{1}{2}\right)^5\right| = \tfrac{1}{224} < \tfrac{1}{200} \text{ as wanted.}$$

c)

$$g(x) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \ldots$$

$$g'(x) = \sum_{n=0}^{\infty}(-1)^n\frac{(2n+1)x^{2n}}{2n+3} = \frac{1}{3} - \frac{3x^2}{5} + \frac{5x^4}{7} - \ldots + (-1)^n\frac{(2n+1)x^{2n}}{2n+3} + \ldots$$